Printed page: 4 **Subject Code: BMIAS0103**

Roll No: | | | | | | | | | | | | | | |

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

M.TECH (Integrated) SEM I- THEORY EXAMINATION (2023-2024)

Subject: ENGINEERING MATHEMATICS - I

Time: 3Hours **Max. Marks:100**

General Instructions:

IMP: Verify that you have received question paper with correct course, code, branch etc.

1. *This Question paper comprises of three Sections -A, B, & C. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*
2. *Maximum marks for each question are indicated on right hand side of each question.*
3. *Illustrate your answers with neat sketches wherever necessary.*
4. *Assume suitable data if necessary.*
5. *Preferably, write the answers in sequential order.*
6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION – A 20

1. Attempt all parts:-

1-a. The rank of matrix $\begin{bmatrix} 0 & -5 & 1 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}$ is : (CO1) 1

(a) 2

(b) 1

(c) 3

(d) None of these

1-b. The Eigen values of a matrix A are 1,2 and 5. The Eigen values of A^2 are : (CO1) 1

(a) 5,1,2

(b) 1,4,25

(c) 2,4,10

(d) None of these

1-c. If $y = Sin\, x$, the value of $y_n(x)$ is : (CO2) 1

(a) $Cos\,(\frac{n\pi}{2} + x)$

(b) $Sin\,(\frac{n\pi}{2} + x)$

(c) $Sin\frac{n\pi}{2}$

(d) None of these

1-d. If $z = \frac{x^2+y^2}{x+y}$ then z is a homogeneous function of degree : (CO2) 1

(a) 2

(b) 1

(c) 0

(a) None of these

1-e. The stationary points of the function $x^2 + y^2 + 6x + 12$ is: (CO3) 1

(a) (-3,0)

(b) (0,3)

(c) (0,0)

(d) None of these

1-f. If $x = u^2 - v^2, y = 2uv$, then the value of the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ is : (CO3) 1

(a) $4(u^2 + v^2)$

(b) $4(u^2 - v^2)$

(c) $(u^2 + v^2)$

(d) None of these

1-g. The value of $\int_0^2 \int_0^2 x^2 dxdy$ is : (CO4) 1

(a) 16/3

(b) 32/3

(c) 3/32

(d) None of these

1-h. The value of integral $\int_0^\infty e^{-x} x^2 dx$ is : (CO4) 1

(a) 2/7

(b) 2

(c) 1

(d) None of these

1-i. 500 is increased by 20% then the new number is : (CO5) 1

(a) 520

(b) 600

(c) 480

(d) None of these

1-j. If Z=52 and ACT=48, then BAT will be equal to : (CO5) 1

(a) 39

(b) 41

(c) 44

(d) 46

2. Attempt all parts:-

2.a. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, find the value of $A^2 - 4A + 5I$? (CO1) 2

2.b. If $f(x, y) = \frac{1}{x^2} + \frac{1}{xy}$, then find value of $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$. (CO2) 2

2.c. Expand e^x by Maclaurine's theorem. (CO3) 2

2.d. Find the value of $\int_0^{\frac{\pi}{2}} sin^3x\, cos^2x\, dx$. (CO4) 2

2.e. If 3 + 5 + 7 + ……..is in A.P., then find 15th term. (CO5) 2

SECTION – B 30

3. Answer any <u>five</u> of the following-

3-a. Determine the value of λ and μ so that the equations $x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu$ have (i) no solution (ii) a unique solution and (iii) infinite many solutions. (CO1) 6

3-b. Find the eigen values and eigen vectors of the matrix, $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. (CO1) 6

3-c. If $u = u\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$. (CO2) 6

3-d. A balloon in the form of right circular cylinder of radius 1.5 m and length 4 m and is surmounted by hemispherical ends. If the radius is increased by 0.01m and the length by 0.05m, find the percentage change in the volume of the balloon. (CO3) 6

3-e. Obtain Taylor's expansion of $tan^{-1}\frac{y}{x}$ about (1,1) up to and including the second degree terms . (CO3) 6

3-f. Establish the relation between Beta and Gamma function $\beta(m, n) = \frac{\lceil m \lceil n}{\lceil m+n}$. (CO4) 6

3-g. If the price of a commodity is raised by 20%, find by how much percent a householder must reduce his consumption of that commodity so as not to increase his expenditure. (CO5) 6

SECTION – C 50

4. Answer any <u>one</u> of the following-

4-a. Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Hence find A^{-1}. Also evaluate $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$. (CO1) 10

4-b. Diagonalize the matrix: $\begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \end{bmatrix}$. (CO1) 10

5. Answer any <u>one</u> of the following-

5-a. If $y = e^{mcos^{-1}x}$, find $y_{n((0)}$. (CO2) 10

5-b. If $u = tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin2u. Hence\ evaluate\ x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$. (CO2) 10

6. Answer any <u>one</u> of the following-

6-a. Use the Jacobian to prove that the functions: $u = x + y - z, v = x - y + z\ and\ w = x^2 + y^2 + z^2 - 2yz$ are functionally dependent. Find the relationship between them. (CO3) 10

6-b. Use Lagrange's method of undetermined multipliers to find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid whose equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. (CO3) 10

7. Answer any <u>one</u> of the following-

7-a. Evaluate $\iint x^2 dx\, dy$, over the region A, where A is the region in the first quadrant bounded by the hyperbola $xy = 16$ and the lines $y = x$, $y = 0$ and $x = 8$. (CO4) 10

7-b. Apply Dirichlet's integral to find the volume and mass contained in the solid region in the first octant of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, if the density at any point is $\rho(x, y, z) = kxyz$. (CO4) 10

8. Answer any <u>one</u> of the following-

8-a. (i) In a certain code language, 'si po re' means 'book is thick', 'ti na re' means 'bag is heavy', 'ka si' means 'interesting book' and 'de ti' means 'that bag'. What should stand for 'that is interesting' in that code language?
(ii) Find the missing term of the given series 3, 5, 9, 17, 33, 65, …?
(iii) A batsman had a certain average of runs for 16 innings. In the 17th innings, he made a score of 87 runs thereby increasing his average by 3. What is his average after 17 innings? (CO5) 10

8-b. (i) A dealer purchased a microwave oven for Rs10000. He offered a discount of 20% on its listed price and still gains 10%. Find the listed price of the microwave oven.
(ii) In a certain code language, '617' means 'sweet and hot', '735' means 'coffee is sweet' and '263' means 'tea is hot'. What should be the code for 'coffee is hot'?
(iii) A average age of 9 members of a club is 29 years. If 2 more persons with the average age of 40 years have become the members of the club, find average age of all the 11 members? (CO5) 10